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Effect of damaged vehicle evacuation on traffic flow with open boundaries

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The effects of the damaged car evacuation (P_{exit}) and the collision (P_{col}) probabilities on the traffic flow behavior of a car accident are investigated in the one-dimensional cellular automaton Nagel-Schreckenberg model, with injecting (α) and extracting (β) rates in parallel dynamics. In this study, we suppose that the car involved in collision is evacuated from the road, with an exit probability P_{exit}. It is found that the behaviors of density, current, and (α,β) phase diagram topology depend strongly on the values of P_{exit} and P_{col}. Indeed, the high-density region shrinks when increasing P_{exit}. Moreover, the critical value $\alpha_c(\beta)$, at which the low-density–high-density transition occurs, increases when increasing P_{col} and/or P_{exit}. Furthermore, the critical value at which the transition high-density–low-density occurs decreases when increasing β and increases with α.

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I. INTRODUCTION

In the majority of countries throughout the world, human life is very much affected by vehicular traffic, which fulfills many passengers' daily needs. But this traffic constitutes a source of serious problems because car accidents are responsible for several losses on human lives and materials. Recently, car traffic problems have been extensively studied and various traffic models have been proposed [1–9]. Several traffic models have been studied within cellular automata (CA) network [4,7,10–12], especially in the case of highway and urban traffics. The effect of maximal velocity on the traffic flow behavior was originally proposed by Nagel and Schreckenberg (NS) [3] and subsequently developed by other authors [4,7,11–14] using several techniques. In CA models, vehicles are considered as particles and the road is modeled as a sequence of cells. Each cell can be empty or occupied by one vehicle. The system evolves in discrete time steps. In most of the models, vehicles move according to parallel dynamics. Furthermore, traffic jams are the major problems in our modern life which often appear in densely populated conglomerations. From intuition, jams result when the outflow is lower than the inflow, while when inflow is lower than the outflow, the cars can move freely. Moreover, car accidents are among the essential causes of congestions in the road. Recently, the occurrence of car accidents has been largely studied within CA models [13–19]. However, to avoid jams on the road when car accidents occur, the seriously damaged cars must be removed from the road.

Our aim in this paper is to study numerically the effect of evacuation and collision probabilities on the traffic flow behavior of the one-dimensional cellular automata NS model with open boundaries in parallel dynamic updates. The effect of the injecting and extracting rates on the traffic flow phase diagram is also examined. However, the model exhibits a variety of (α,β) phase diagrams depending on the values of the model parameters.

The paper is organized as follows. In the following section, we define the conditions of the occurrence of car accidents and we present the model. Section III is reserved for the main results and discussion. Finally, a conclusion is drawn in Sec. IV.

II. MODEL

As we have said in Sec. I, the main idea of this work is to study the effect of the combination of the collision (P_{col}) and removing (P_{exit}) probabilities on density and current flux in the deterministic NS model. When an accident occurs with a probability P_{col} at a position $x(i,t)$ at time t, the vehicle involved in the collision leaves the road with a probability P_{exit} immediately at this position. The evacuation takes place where the accident took place. Clearly, the probabilities P_{col} and P_{exit} are defined as follows. These two parameters are extrinsic and the same for all locations. However, at each location of the accident, we randomly draw a number between 0 and 1: if this number is lower than P_{exit} (respectively, P_{col}), the damaged vehicle leaves the road (respectively, causes an accident), if not, the vehicle continues its way. The placing of the random number differs from one site to another. In fact, P_{col} is the probability with which a vehicle may cause a collision with the car ahead if the conditions cited in the Ref. [13] are verified. Practically, this probability can depend on the state of the driver and/or the road as it may depend on the state of the vehicle. Theoretically, P_{col} is a given parameter because it is difficult to define all statements cited above. P_{exit} also is an external parameter but in practice it may depend on the state of the vehicle after collision. Finally, for the given values of the independent parameters P_{exit} and P_{col}, the evacuation of a car involved in the accident is proportional to the product $P_{col}P_{exit}$. In other words, for a fixed value of P_{col}, P_{exit} can take all values between 0 and 1.

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Let us consider a road of one-dimensional lattice of length L. Each lattice site is either empty or occupied by a car of velocity $v=0,1,2,\ldots,V_{\max}$. We denote by $x(i,t)$ and $v(i,t)$, respectively, the position and the velocity of the ith car at time t and $d(i,t)=x(i+1,t)-x(i,t)-1$ the number of empty sites in front of a car which is called hereafter the gap.

The system update is performed in parallel for all cars in open boundaries according to the following four rules:

(i) Acceleration: if the speed of car is lower than $V_{\max}$, it is increased by 1.

(ii) Deceleration: if the speed of ith car is larger than $d(i,t)$, then it is reduced to $d(i,t)$.

(iii) Randomization: the speed of a moving car is decreased randomly by one unit with a braking probability p.

(iv) Car motion: car is moved forward according to its new speed determined by the above three rules.

In the basic NS model, car accidents will not occur because the second rule is designed to avoid collisions. The safety distance of driver is respected in the driving scheme. However, in real traffic, car accidents often occur because of careless drivers who have tendencies to drive as fast as possible and increase the safety velocity given in the second rule by one unit with a probability P_{col}, which is assumed to be independent of the car and the time. Next, it will arrive at the position of the moving car ahead. If the car ahead suddenly stops, the collision between two neighboring i and $i+1$ cars occurs [13,14]. This collision concerns the states of two neighboring cars at different time steps; therefore, they are correlative, both spatially and temporal. The accident occurred when a vehicle that precedes another stops abruptly. If the driver of the second vehicle is not careful, he may cause a collision with the vehicle ahead. We note that in this study, we are mainly interested in the second vehicle which caused the accident. But even when we evacuated the two vehicles which collide, the results are qualitatively unchanged. The collision between two neighboring cars i and $i+1$ will then exist at time $t+1$ with a probability P_{col} if the following conditions are simultaneously satisfied [13]:

$$\text{C1:}\ \ 0 \le d(i,t) \le V_{\max},$$

$$\text{C2:}\ \ v(i+1,t) > 0,$$

$$\text{C3:}\ \ v(i+1,t+1) = 0.$$

Although the probability for car accidents has been studied in the basic model [13,14], car accidents do not really happen; it is the NS model that is used. In the numerical simulation results, the above three necessary conditions (C1, C2, and C3) are only regarded as a dangerous situations and an indicator of collisions allows us to know the number of accidents on the road. More precisely, the car accident algorithm is divided into two independent parts: (1) check on car accident and (2) forward movement:

(1) If the three above conditions C1, C2, and C3 are simultaneously verified, an indicator indicates that accident between cars happens with a probability P_{col}. To improve the model cited above [13], the damaged cars involved in the collisions are evacuated with a probability P_{exit}. However, for $P_{exit}=0$, no car leaves the road and then this case is equivalent to the basic NS model [3,13,14].

(2) The configuration of the system at the time $(t+1)$ is computed from that of time t according only to the four subrules of the NS model regardless of accidents. Ultimately, no vehicles blocking traffic. The vehicle that collides is either evacuated from the road with a probability P_{exit} or has run continuously like other vehicles.

Besides, we suggest that the car causing the collision will be removed from the road with an exit probability P_{exit}. In reality, this probability depends on the state of the vehicle involved in the collision. In view of the loss of information about the state of such a vehicle, hereafter, the exit probability is introduced as a stochastic parameter $(0 \le P_{exit} \le 1)$.

We note that even when P_{col} is constant, the probability of occurrence of accident (rated P_{ac} in Ref. [13]) is a dynamic variable and depends on the state of traffic flow which makes the analysis more complex. But our main interest in this work is to study the effect of extrinsic parameters P_{col} and P_{exit} on the density and current flow. We noticed that in most of the previous works [13,14], P_{col} was used without specifying its variation domain.

III. RESULTS AND DISCUSSION

We simulate one-lane traffic using the deterministic NS model with a one-dimensional lattice of length $L=1000$ sites and $V_{\max}=5$ with open boundary conditions. The model parameters are the injecting rate α, the extracting rate β, and the two stochastic parameters P_{exit} and P_{col}.

We start with random positions and velocities of cars at the initial configurations. Next, we update the individual vehicle velocities and positions in accordance with the NS-update rules in parallel dynamics. The results are obtained by averaging over 80 initial configurations and 2×10^3 time steps after discarding 2×10^4 initial transient states.

First, we investigate the influence of the exit probability P_{exit} on the average density of cars in the road for a given value of collision probability P_{col}. However, Fig. 1(a) shows the variation of the density ρ as a function of the injecting rate α for $P_{exit}=0.1$, $P_{col}=10^{-3}$, and for various values of extracting rate β. For $\alpha<\beta$, the system is in the low-density state, there is no collision because the three conditions C1, C2, and C3 cannot be satisfied simultaneously; especially the condition C1 because the gaps are larger than $V_{\max}$. While, at high value of injecting rate α, the system is in the high-density state, where the collision of particles occurs with a probability P_{col} which leads to the evacuation of vehicles from the road with a probability P_{exit}. However, when increasing P_{exit}, the evacuation becomes more important and then the density at the steady state decreases. Moreover, the system exhibits a first-order transition at $\alpha>\beta$ because of the disorder induced by car accidents. This result is in good agreement with Refs. [20,21]. Since the order parameter here is the density, we have a similar situation to the behavior of the order parameter in finite-size systems [20]. We identify the first-order transition for the system size L by the appearance of a peak in the derivative of $\rho(\alpha)$ function with respect

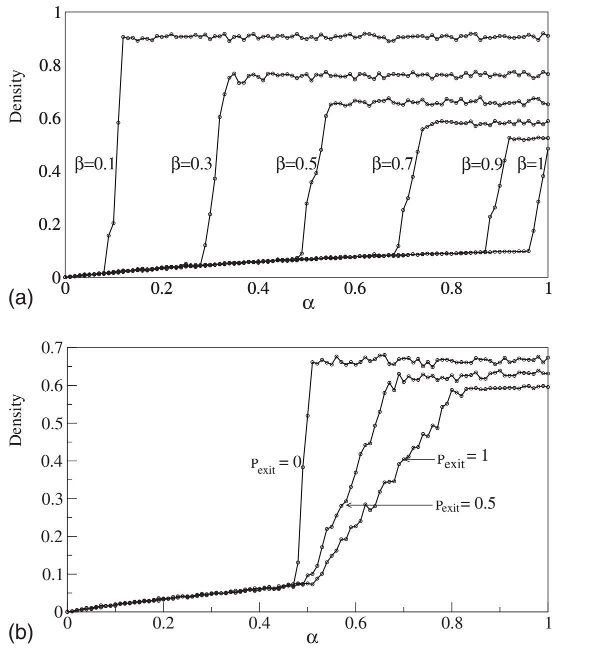

FIG. 1. Variation of density ρ as a function of injecting rate α for $P_{col}=10^{-3}$. (a) $P_{exit}=0.1$ and for different values of extracting rate β. (b) $\beta=0.5$ and for three values of P_{exit}.

to α. Otherwise, the derivative will undergo a jump at the second-order transition.

For low values of P_{exit} and P_{col}, similarly to the results found in Refs. [20,21], Fig. 1(b) shows the presence of a first-order transition from low-density phase (LDP) to high-density phase (HDP) when increasing α. While for large values of P_{exit} and P_{col}, the transition is of second-order-like. Besides, for large values of injecting rate, the saturation value of density decreases when increasing P_{exit}. In fact, the case of $P_{exit}=0$ is studied by several previous works in which the authors do not take into account the effect of the collision probability on the traffic flow behavior, such as current and velocity [13,14]. Consequently, for $P_{exit}=0$, the behavior of density as a function of the injecting rate α undergoes a sharp low-high density transition. This case corresponds exactly to the result obtained in the standard NS model [20,21]. However, our main task in this paper is to make in relation the event of car accidents with the traffic flow behavior. When increasing P_{exit}, the evacuation of damaged vehicles increases, then the effective extracting rate given by $\beta_{eff}=\beta+\beta_{exit}$ increases. β_{exit} is an extracting rate which is proportional to the product $P_{col}P_{exit}$. It is clear that the injecting rate α at which the transition low-high density occurs increases when increasing P_{exit}.

The study of the current behavior as a function of injecting rate α, for various values of β and for $P_{exit}=0.1$ and $P_{col}=10^{-3}$, is given in Fig. 2(a). The current is defined as $J=\rho V_m$, where ρ and V_m are, respectively, the density and the average velocity of vehicles calculated in the steady state. In low-density phase, the current increases with the injecting rate α. While when increasing α, the current undergoes a saturation value [20,21]. We note that the saturation current increases when increasing β [Fig. 2(a)] and/or P_{exit} [Fig. 2(b)].

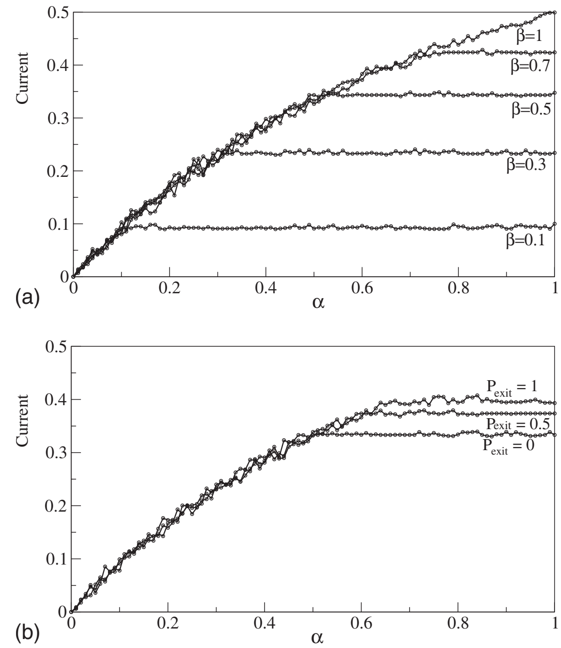

FIG. 2. Variation of current $\langle J\rangle$ vs the injection rate α for $P_{col}=10^{-3}$. (a) $P_{exit}=0.1$ and for different values of extracting rate β and (b) for $\beta=0.5$ in three following cases: (1) without removing cars, (2) $P_{exit}=0.5$, and (3) $P_{exit}=1$.

Phase diagrams presented in Figs. 3(a) and 3(b) summarizes the most important behavior of such a model. However, Fig. 3(a) shows that the low-density–high-density phase transition occurs at $\alpha>\beta$ for $P_{exit}>0$ and at $\alpha=\beta$ for $P_{exit}=0$, which means that the region of the high density shrinks with increasing P_{exit}. Furthermore, for $\alpha=1$ the LDP-HDP transition takes place at $\beta=\beta_0<1$ for sufficiently large values of P_{exit}. For example, for $P_{exit}=1$, $\beta_0=0.8$. While, for suffi-

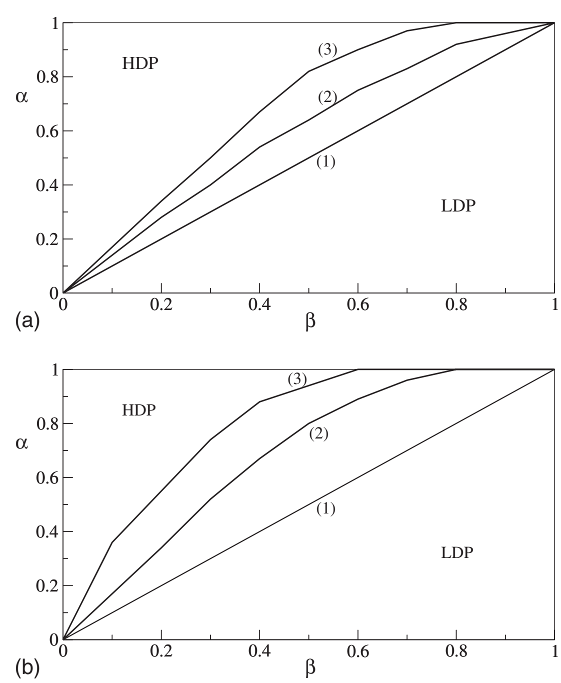

FIG. 3. Phase diagram (α,β) planes for (a) $P_{col}=10^{-3}$ and (b) $P_{col}=2\times10^{-3}$. (1) $P_{exit}=0$, (2) $P_{exit}=0.5$, and (3) $P_{exit}=1$.

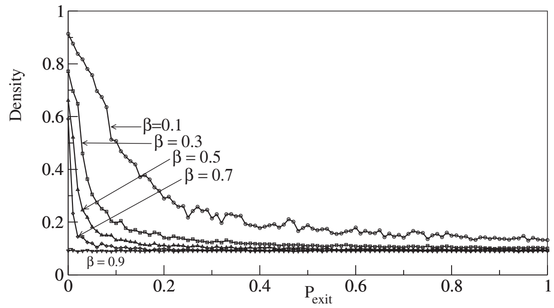

FIG. 4. Variation of density ρ as a function of removing rate P_{exit} for P_{col}=0.1 and α=0.8 and for several values of β.

ciently small values of P_{exit}, such a transition is closed to β=1. Moreover, Fig. 3(b) shows that the value of β_0 decreases when increasing P_{col}.

The previous results [20,21] showed that the first-order transition in density occurs for α=β, which corresponds to a straight line in the (α,β) phase diagram, while in our case, the transition line is not straight but curved (α>β). This curvature is due to the disorder or disturbance of the system caused by the collision of cars. Such a result has been already revealed where the disorder breaks the symmetry between sites occupied and empty ones [21].

Furthermore, in order to provide more information concerning the effect of the exit probability P_{exit} on density, we plot $\rho(P_{exit})$ as shown in Fig. 4 for several values of β. In this figure, we consider the case of a relatively important value of accident probability (P_{col}=0.1). The average density of cars decreases rapidly when increasing P_{exit} since the evacuation becomes important and remains constant for large value of P_{exit}. Besides, when the extracting rate β is sufficiently large, the density is very low and remains constant with P_{exit}.

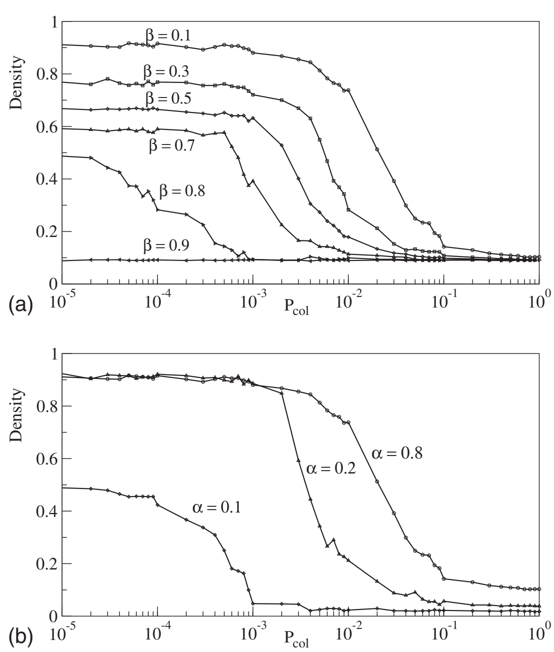

FIG. 5. Average density behavior as a function of P_{col} in the case where P_{exit}=0.5 (a) for α=0.8 and for several values of β and (b) for β=0.1 and for different values of α.

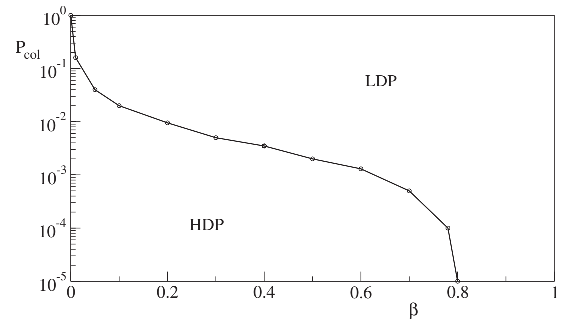

FIG. 6. Phase diagram (P_{col},β) plane for P_{exit}=0.5 and α=0.8.

Now we examine, in Fig. 5, the behavior of density as a function of P_{col} for P_{exit}=0.5. It is found that the density decreases when increasing P_{col}, reaching its lower value for $P_{col}>P_{col_0}$, for different values of β [Fig. 5(a)] and different values of α [Fig. 5(b)]. P_{col_0} decreases when increasing β and/or decreasing α. This result is summarized in the phase diagram given in Fig. 6, in which the system exhibits high-density–low-density transition. High-density phase is favored when decreasing P_{col}. Besides, HDP-LDP transition occurs at β=α when P_{col}=0.

Finally, Figs. 7(a) and 7(b) give a clear idea about the space-time distribution of car density in both sides of the

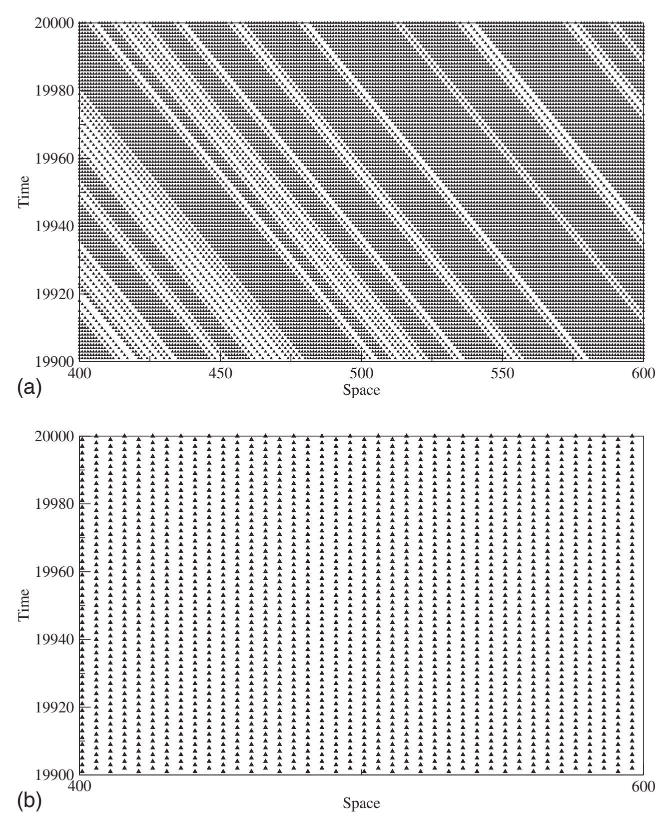

FIG. 7. Space-time structure for α=0.8, P_{exit}=0.5, and P_{col} =10^{-2} in two following cases: (a) β=0.1 and (b) β=0.8. Each horizontal row of full triangles represents the instantaneous position of cars moving right, while the successive rows indicate the position of the same cars at successive time steps.

dividing line of HDP-LDP transition shown in Fig. 6. Indeed, for α=0.8, P_{exit}=0.5, and P_{col}=10^{-2}, Figs. 7(a) and 7(b) show the repartition of cars in space and time, respectively, for β=0.1 and β=0.8. Each horizontal row of full triangles represents the instantaneous position of cars moving right while the successive rows indicate the position of the same cars at successive time steps. For a relatively important probability of accidents and for an intermediate probability of removing damaged cars, a small value of extracting rate β results in a jamming situation in the road. On the other hand, a large value of β allows a free flow, the cars can move forward freely in the low-density region without formation of jams.

IV. CONCLUSION

In this paper, we have numerically studied the effects of injecting and extracting rates on car accidents, with evacuation of damaged vehicles, on the behavior of average density and current flow in the road. The results are obtained using deterministic NS model in parallel dynamics update. We have shown that the behaviors of density and current depend strongly on the collision and evacuation probabilities. Besides, the (α, β) phase diagrams exhibit several kinds of different topologies, especially the curvature of the low-high density transition.

[1] D. Chowdhury, L. Santen, and A. Schadschneider, Phys. Rep. **329**, 199 (2000).
[2] D. Helbing, Rev. Mod. Phys. **73**, 1067 (2001).
[3] K. Nagel and M. Schreckenberg, J. Phys. I **2**, 2221 (1992).
[4] A. Schadschneider and M. Schreckenberg, J. Phys. A **26**, L679 (1993).
[5] H. Ez-Zahraouy, Z. Benrihane, and A. Benyoussef, Int. J. Mod. Phys. B **18**, 2347 (2004).
[6] M. Schreckenberg, A. Schadschneider, K. Nagel, and N. Ito, Phys. Rev. E **51**, 2939 (1995).
[7] K. Nagel, D. E. Wolf, P. Wagner, and P. Simon, Phys. Rev. E **58**, 1425 (1998).
[8] P. Wagner, K. Nagel, and D. E. Wolf, Physica A **234**, 687 (1997).
[9] A. Mhirech, A. Isamili Alaoui, and H. Ez-Zahraouy, Can. J. Phys. **87**, 105 (2009).
[10] M. Fukui and Y. Ishibashi, J. Phys. Soc. Jpn. **62**, 3841 (1993).
[11] T. Nagatani, J. Phys. A **26**, L781 (1993).
[12] H. Ez-Zahraouy, K. Jetto, and A. Benyoussef, Eur. Phys. J. B **40**, 111 (2004).
[13] N. Boccara, H. Fuks, and Q. Zeng, J. Phys. A **30**, 3329 (1997).
[14] X. Q. Yang, Y. Q. Ma, and Y. M. Zhao, Phys. Rev. E **70**, 046121 (2004).
[15] X. Q. Yang, W. Zhang, K. Qiu, and Y. M. Zhao, Phys. Rev. E **73**, 016126 (2006).
[16] X. Yang, Y. Ma, and Y. Zhao, J. Phys. A **37**, 4743 (2004).
[17] X. Yang, Y. Ma, and Y. Zhao, Mod. Phys. Lett. B **16**, 333 (2002).
[18] X. Yang and Y. Ma, J. Phys. A **35**, 10539 (2002).
[19] A. Parmeggiani, T. Franosch, and E. Frey, Phys. Rev. Lett. **90**, 086601 (2003).
[20] A. Benyoussef, H. Chakib, and H. Ez-Zahraouy, Eur. Phys. J. B. **8**, 275 (1999); M. N. Barber, *Finite Size Scaling*, edited by C. Domb and J. Lebowitz (Academic Press, London, 1983), Vol. 8.
[21] M. Bengrine, A. Benyoussef, H. Ez-Zahraouy, and F. Mhirech, Phys. Lett. A **253**, 135 (1999).